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Reg. No.____________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA202: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1. a. Given that $f(x)=\frac{k}{2^x}$ is a probability distribution of a random variable that can take on the values $x=0,1,2,3$ and 4, find k. Find the cumulative distribution function. (7)

   b. If 6 of the 18 new buildings in a city violate the building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch

   i) none of the new buildings that violate the building code

   ii) one of the new buildings that violate the building code

   iii) at least two of the new buildings violate the building code (8)

2. a. Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np=\lambda$ a constant. (7)

   b. Find the value of k for the probability density $f(x)$ given below and hence find its mean and variance where

   $$f(x)=\begin{cases} kx^3 & 0<x<1 \\ 0 & otherwise \end{cases}$$

   (8)

3. a. A random variable has normal distribution with $\mu=62.4$. Find it's standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 (7)

   b. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with the parameter 50 days. Find the probability that such a camera will

   i) have to be reset in less than 20 days

   ii) not have to be reset in at least 60 days. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4. a. Use Fourier integral to show that $\int_0^\infty \frac{\cos x\omega+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases} 0 & if\ x<0 \\ \pi/2 & if\ x=0 \\ \pi e^{-x} & if\ x>0 \end{cases}$ (7)

   b. Represent $f(x)=\begin{cases} x^2 & 0<x<1 \\ 0 & x>1 \end{cases}$ as a Fourier cosine integral. (8)

5. a. Find the Fourier transform of $f(x)=\begin{cases}1 & if\ |x|<1\\ 0 & otherwise\end{cases}$ (7)

b. Find the Laplace transforms of the following

i) $\cos t - t \sin t$ ii) $4t\,e^{-2t}$ (8)

6. a. Find the inverse Laplace transform of the following

i) $\dfrac{2s+1}{s^2+2s+5}$ ii) $\dfrac{(2s-10)}{s^3}e^{-5s}$ (8)

b. Solve $y''+2y'+5y=25t$, $y(0)=-2$, $y'(0)=-2$ using Laplace transforms (7)

PART C (MODULES V AND VI)

Answer two full questions.

7. a. Solve $f(x)=x-0.5\cos x=0$ near $x=0$ by fixed point iteration method. (7)

b. Solve $f(x)=2x-\cos x=0$ by Newton Raphson's method (7)

c. Find $f(9.2)$ from the values given below by Lagrange's interpolation formula

x	8	9	9.5	11
$f(x)$	2.197225	2.251292	2.397895	2.079442

(6)

8. a. Given $(x_j, f(x_j)) = (0.2, 0.9980), (0.4, 0.9686), (0.6, 0.8443), (0.8, 0.5358), (1,0)$, find $f(0.7)$ based on 0.2, 0.4, and 0.6 using Newton's interpolation formula. (10)

b. Solve $10x_1+x_2+x_3=6$, $x_1+10x_2+x_3=6$, $x_1+x_2+10x_3=6$ by Gauss-Seidel iteration method starting at $x_1=1$, $x_2=1$ and $x_3=1$ correct to 4 digits. (10)

9. a. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with 4 subintervals by Simpson's rule and compare it with the exact solution. (7)

b. Solve $y'=y$, $y(0)=1$ by Euler method to find $y(1)$ with $h=0.2$ (7)

c. Solve $y'=1+y^2$, $y(0)=0$ by fourth order Runge-Kutta method with $h=0.1$, 5 steps. (6)

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